Niching Evolution Strategy with Cluster Algorithms

O. Aichholzer, F. Aurenhammer, B. Brandstätter, T. Ebner, H. Krasser, Ch. Magele

Abstract— **In most real world optimization problems one tries to determine the global among some or even numerous local solutions within the feasible region of parameters. One the other hand, it could be worth to investigate some of the local solutions as well. Therefore, a most desirable behaviour would be, if the optimization strategy behaves globally and yields additional information about local minima detected on the way to the global solution. In this paper a clustering algorithm has been implemented into an Higher Order Evolution Strategy in order to achieve these goals.**

Keywords—**Stochastic Optimization, Evolution Strategies, Clustering Methods**

I. Introduction

Stochastic optimization methods can be successfully applied to problems, where there is no or very little knowledge about the behaviour of the objective function, about the presence of local minima or the distribution of feasible and nonfeasible regions within the multi-dimensional parameter space. A substantial difference to deterministic optimization methods consists of the fact that stochastic methods take advantage of "random decisions". Evolution Strategies (ES), an important group among stochastic optimization methods, rely on a simplified model of biological evolution and imitate several statistical processes acting on or within populations. Among these processes there is *reproduction*, *mutation*, *competition* and *selection*. *Reproduction* is an obvious property of all life and so is *mutation*. Both features can already be found in the most simple ES, the (1+1) Evolution Strategy [3]. Higher order $(\mu/\rho, \lambda)$ ES [3] introduce more than two competing individuals and set up a *population* of appropriate size constrained to a finite arena, so that *competition* and *selection* become the inescapable consequences [2]. Strategies of that kind are very likely to find the global solution of a given problem, if the strategy parameters are chosen in an appropriate way [1]. Additionally a lifetime parameter τ, specifying the number of generations an individual is allowed to survive within the population, can be defined.

Having a closer look at the behaviour of higher order ES on their way to the final solution, it can be recognized, that they pass by regions with local solutions without giving notice. Therefore, these solutions remain unknown to the user. This disadvantage can be overcome by introducing more than one population into the strategy. A control process guides some of these populations temporarily into some of the local minima and informs the user about these solutions. Nevertheless, the overall behaviour of the ES must remain globally. Several of these "niching methods" have been presented recently [6], [?]. In this paper a cluster algorithm based on a complete linkage was implemented to detect κ individual sub-populations temporarily gathering around many local solutions.

A multi-modal, two dimensional test problems, a modified Rosenbrock function and the 8 parameter SMES benchmark problem [1] are solved using this approach.

Manuscript received June 4, 2000

Work partially supported APART and by the Austrian "Fonds zur Foerderung der wissenschaftlichen Forschung" under the grant No. F003

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II. $[\kappa(\mu(\tau)/\rho, \lambda)]$ Evolution Strategy

A $[\kappa(\mu(\tau)/\rho, \lambda)]$ ES sets up κ sub-populations, where μ is the average number of parents per subpopulation. Each parent survives at most for τ generations. The total number of individuals therefore is $\kappa * \mu * \tau$. In each generation, ρ out of all mating parents take part in the recombination process to produce $\kappa * \lambda$ children. All children have then to undergo the mutation operation [1]. The $\kappa * \mu$ best children then replace the oldest parents and the iteration process continues.

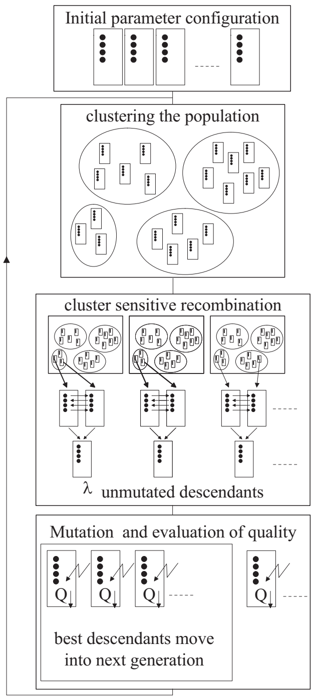

Fig. 1. Higher Order $[\kappa(\mu(\tau)/\rho, \lambda)]$ Evolution Strategy

A. *Hierachical Clustering the Population*

To apply clustering we treat every parameter configuration as a single object. Grouping these objects together is done by complete linkage, or C-link for short, an agglomerative hierarchical clustering method [4] ,[5]. This class of methods starts with n single objects (parameter configurations), all seen as their own clusters. The two clusters to be merged in the next step are chosen in a way that the dissimilarity of the resulting cluster is minimized among all possible pairs. After $n-1$ steps, all objects form in a single final cluster the root of the so-called cluster tree. For simple example see Figure 2.

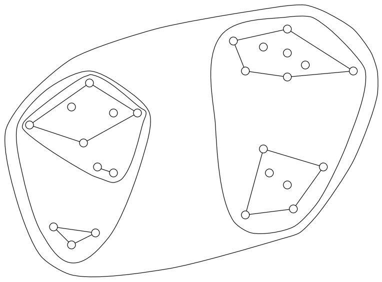

Fig. 2. The nine top level clusters of a hierarchical cluster tree.

B. Cluster Sensitive Recombination

Since, in general, the optimization parameters are floating point numbers, *recombination* is performed by *arithmetic crossover*. Two new descendant configurations are set up from two parental ones (1).

$$\begin{aligned}\mathbf{x}_{descendant,1} &= a\mathbf{x}_{parent,1} + (1-a)\mathbf{x}_{parent,2}\\ \mathbf{x}_{descendant,2} &= a\mathbf{x}_{parent,2} + (1-a)\mathbf{x}_{parent,1}\end{aligned} \quad (1)$$

The factor a is chosen randomly from $[0.8 \cdots 1.2]$. The candidates for recombination are chosen in the following way.

- Choose one of the κ clusters randomly
- Choose one candidate from this cluster, taking its fitness into account (roulette wheel) [1]
- Choose the second candidate, whereas members from the current cluster are more probable then members from remote clusters

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III. Results

A. Modified Rosenbrock Function

The unimodal Rosenbrock function

$$f_{rosen}(\mathbf{x}) = 100(x_2 - x_1 * x_1)^2 + (1-x_1)^2 \quad (2)$$

was modified to show three distinct minima (3).

$$\begin{aligned}f_{rosen,mod}(\mathbf{x}) = f_{rosen}(\mathbf{x}) &- 50((x_1+1)^2 + (x_2-1)^2)\\ &-10((x_1-1.5)^2 + (x_2-2.5)^2) + e^{2x_2-5}\end{aligned} \quad (3)$$

Fig. 3 shows the initial population clustered into five subpopulations.

Form now on the *cluster sensitive recombination* is applied, which leads to an intermediate state as presented in

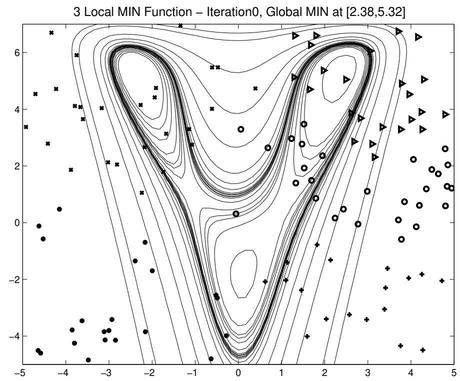


Fig. 3. Modified Rosenbrock function, initial population

Fig. 5. If this situation can be detected, all possible solutions of the problem have been found. One way to do so is to assess *diameter curves* in the due course of the optimization. At each iteration step the diameters of the largest cluster of all possible clusters (from 1 to the κ) is evaluated.

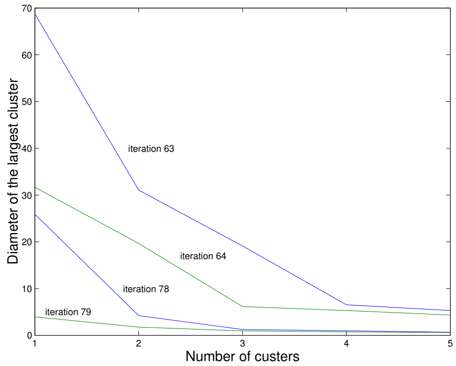


Fig. 4. Diagrams of the largest cluster diameters at different iteration steps

Fig. 4 shows a selection of such curves at 4 interesting time instances. Looking at the sequence with respect to the largest diameters it can be seen that something significantly happens after generation 63 (=iteration 63) and generation 78. In fact this is where the strategy reduces the three clusters to two and the two clusters to one, respectively.

If the Evolution Strategy is continued, a final population as plotted in Fig. 6 is reached, where the stochastic optimization process terminates. It can be seen that the Evolution Strategy guides its population(s) into the global solution.

B. 8 Parameter SMES Optimization Problem

SMES (Superconducting Magnetic Energy Storing) systems consisting of a single superconducting solenoidal coil offer the opportunity to store a significant amount of energy in magnetic fields in a fairly simple and economical way and can be rather easily scaled up in size. However, such arrangements usually suffer from their remarkable stray field. A reduction of the stray field can be achieved if a second solenoid is placed outside the inner one, with a current

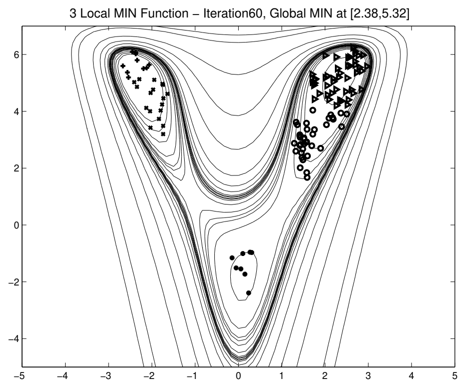


Fig. 5. Modified Rosenbrock function, population spread of three minima

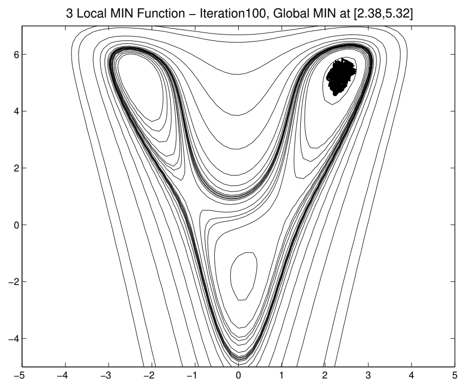
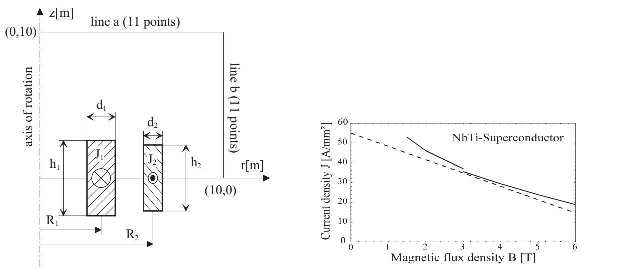


Fig. 6. Modified Rosenbrock function, final population

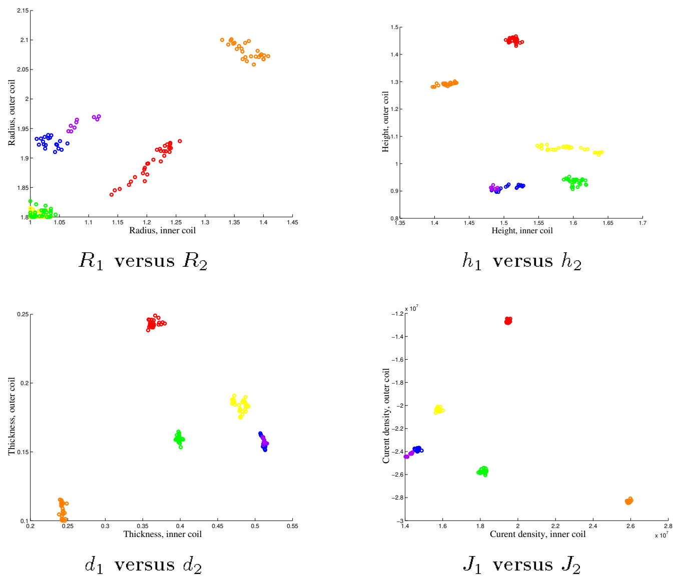
flowing in the opposite direction (Fig. 7). A correct design of the system should then couple the right value of energy to be stored with a minimal stray field (Multi Objective Optimization [1]). The full description of the problem can be found in [7]. The optimal design is not an easy task because, besides usual geometrical constrains, there is a material related constraint: the given current density and the maximum magnetic flux density value on the coil, must not violate the superconducting quench condition which can be well represented by a linear relationship shown in Fig. 7. This restriction leads to a large infeasible region within the parameter space specified by the box constrains.



SMES device   Critical Curve of the Superconductor

Fig. 7. Configuration of the SMES device.

This objective function is set up using fuzzy membership functions [1].



R_1 versus R_2 h_1 versus h_2

d_1 versus d_2 J_1 versus J_2

Fig. 8. SMES device: Intermediate Population

The $[\kappa(\mu(\tau)/\rho,\lambda)]$ Evolution Strategy is applied to find acceptable solutions for this problem. The number of clusters is fixed with κ equals 6. Fig. 8 shows an intermediate population with 6 distinct clusters. Starting a deterministic Quasi Newton Strategy from the locally best configuration of the 6 clusters leads to the results shown in Tab. I. All solutions have acceptable values for the energy and the stray field, although all solutions differ quite significantly from each other.

TABLE I

Some local Solutions of the SMES Problem.

	c1	c2	c3	c4,c5	c6
R_1	1.27	1.43	1.00	1.04	1.02
R_2	1.80	2.04	1.80	1.88	1.80
$\frac{h_1}{2}$	1.40	1.30	1.57	1.39	1.54
$\frac{h_2}{2}$	1.80	1.52	1.06	0.93	0.95
d_1	0.38	0.252	0.488	0.53	0.404
d_2	0.25	0.105	0.180	0.159	0.157
J_1	20.67	26.97	15.91	15.01	18.50
J_2	-11.91	-27.91	-20.46	-24.25	-25.61
$Energy$	179.95	178.84	180.57	178.78	179.41
B_{Stray}	15.85	168.54	357.83	341.13	361.80

IV. Conclusion

Evolution Strategies are already very powerful and still easy-to-use methods to solve complex, multimodal optimization problems. Using proper strategy parameters increases the global behaviour of these methods. Applying a *cluster sensitive recombination*, which requires very little modification to a standard $\mu/\rho,\lambda)]$ Evolution Strategy, allows to identify local solutions (if such exist) during the iteration process. These local solution can then be compared to the best solution. This enables the user to choose among a set of solutions rather than being forced to accept the best one. The increasing number of function calls can well be justified by the number of solutions obtained. Nevertheless, further investigations will be made to speed up the iteration process and to find reliable methods to automatically and dynamically identify the number of cluster present in the parameter space.

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